$$\boxed{\frac{dq}{dt} = F - \frac{1}{\rho}\nabla P}$$

This is known as Euler's equation of motion at all Point of the Perfect fluid.
The dissipative force are not considered

$$\Rightarrow \frac{dq}{dt} = F - \frac{1}{\rho}\nabla P$$

$$\Rightarrow \left(\frac{\partial}{\partial t} + q\nabla\right)q = F - \frac{1}{\rho}\nabla P$$

$\frac{d}{dt} = \frac{\partial}{\partial t} + q\nabla$ or $\frac{\partial q}{\partial t} + (q\nabla)q = F - \frac{1}{\rho}\nabla P$

or $$\frac{\partial q}{\partial t} + \nabla\left(\frac{1}{2}q^2\right) - q \operatorname{curl} q = F - \frac{1}{\rho}\nabla P$$

$$\Rightarrow \boxed{\frac{\partial q}{\partial t} + \nabla\left(\frac{1}{2}q^2\right) + w \times q = F - \frac{1}{\rho}\nabla P}$$

Since $w = \nabla \times q$

Known as Lamb's Hydrodynamical eq^n^. this eq^n^ is non-linear eq^n^ due to the Convective term $(q\cdot\nabla)q$

— x — x —

§ 3.5 / 82 E.E:- Consider the body forces are conservative and the flow is of Potential kind it follows that ∃ scalar function Ω and ϕ such that

$$F = -\nabla\Omega \qquad q = -\nabla\phi$$

this is due to convective — this is due to P.K.

$$\frac{\partial}{\partial t}(-\nabla\phi) + (q\cdot\nabla)q = -\Omega - \frac{1}{\rho}\nabla P$$

$$\Rightarrow -\nabla\frac{\partial\phi}{\partial t} + \nabla\left(\frac{1}{2}q^2\right) = -\nabla\Omega - \frac{1}{\rho}\nabla P$$

to time then

$$\nabla\cdot(\rho q) = 0$$

For a non- homogeneous incompre ssible fluid the density of the fluid Particle remains contant through -out the entire region.

i.e. density remains contant through the in direction

$$\rho(\nabla q) = 0$$

$$\Rightarrow \nabla q = 0$$

The quantity divergent the rate of Volum expansion of a fluid element. It may be (∇q) called dilatation or expansion Physically represent the vector having zero divergence is said to be solenoidal

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Q.(i) State and Prove Eq^n of continuity in vector form

(ii) obtain a kinemetric relation between the velocity and the density of the fluid

(iii) Prove that the countinuity is satisfied a Solenoidal.

§ 3.5 / 82 Integration of Euler's Equation:-

The Euler's equation of motion is Given

$$\frac{\partial q}{\partial t} + \nabla\left(\frac{1}{2}q^2\right) - q \times \text{curl}\, q = F - \frac{1}{\rho}\nabla P \quad \text{--- (1)}$$

Consider the body forces are conservative and that the flow is of the Potential kind then there exist scalar functions Ω and ϕ, such that

$$F = -\nabla\Omega, q = -\nabla\phi \quad \text{--- (2)}$$

From (1) And (2) we have

$$\frac{\partial q}{\partial t} + \nabla\left(\frac{1}{2}q^2\right) - q \times \text{curl}\, q = -\nabla\Omega - \frac{1}{\rho}\nabla P$$

or $$\frac{\partial q}{\partial t} - q \times \text{curl}\, q = -\nabla\left[\Omega + \int \frac{dP}{\rho} + \frac{1}{2}q^2\right]$$

or $$\frac{\partial}{\partial t}(-\nabla\phi) - q \times \text{curl}\, q = -\nabla\left[\Omega + \int \frac{dP}{\rho} + \frac{1}{2}q^2\right]$$

or $$\nabla\left[-\frac{\partial\phi}{\partial t} + \Omega + \int \frac{dP}{\rho} + \frac{1}{2}q^2\right] = q \times \text{curl}\, q \quad \text{--- (3)}$$

I Irrotational flow:-

$$\underline{\omega} = \text{curl}\, q = 0 \text{ then eq}^n \text{ (3)}$$

$$\nabla\left[-\frac{\partial\phi}{\partial t} + \Omega + \int \frac{dP}{\rho} + \frac{1}{2}q^2\right] = 0$$

By Integrating, we have

$$-\frac{\partial\phi}{\partial t} + \Omega + \int \frac{dP}{\rho} + \frac{1}{2}q^2 = \chi(t) \quad \text{--- (4)}$$

Where $\chi(t)$ is arbitray function equation (4) reduces

$$-\frac{\partial\phi}{\partial t} + \Omega + \int \frac{dP}{\rho} + \frac{1}{2}q^2 = \text{constant} \quad \text{--- (5)}$$

known as Bernoulli's eq^n for unsteady irrotational flows.

$$\boxed{x = \frac{x_0}{t_0} t, \; y = y_0 e^{t - t_0}, \; z = z_0} \quad \text{—— (1)}$$

Hence the path lines are given by

Let the fluid particle x_0, y_0, z_0 passing through a fixed point at (x_1, y_1, z_1) along an instant of time t $= T$ such that $t_0 \le T \le t$

$$x_1 = \frac{x_0}{t_0} T$$

$$y_1 = y_0 e^{T - t_0} \quad \text{—— (2)}$$

$$z_1 = z_0$$

Where T is the parameter

From (1) and (2) we have

$$\boxed{x = \frac{x_1}{T} t, \; y = y_1 e^{t - T}, \; z = z_1}$$

This is known as Eq^n. to streaklines

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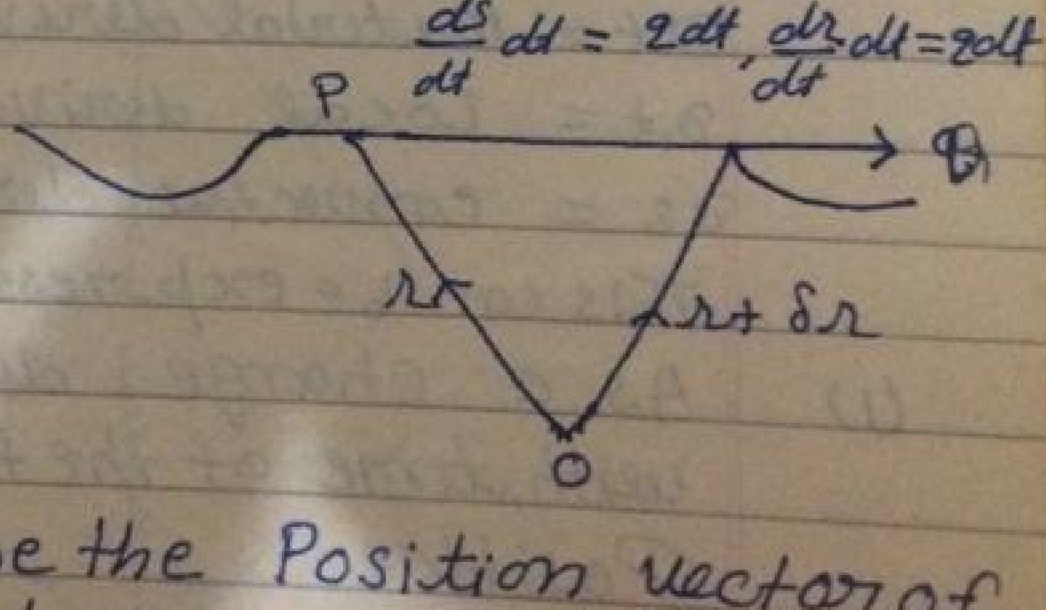
<u>Combination, Local, Convective and Material derivatives:-</u>

$$\frac{ds}{dt} dt = q\,dt, \; \frac{dr}{dt} dt = q\,dt$$



Consider r be the position vector of the point P at an instant of time t and $r + dr$ be the position vector of the point Q at an instant of time $t + dt$

$$\nabla \left\{ -\frac{\partial \phi}{\partial t} + \Omega + \frac{1}{2} q^2 + \int \frac{d\phi}{\rho} \right\} = 0$$

By integlating, we have

$$\Rightarrow \quad -\frac{\partial \phi}{\partial t} + \Omega + \frac{1}{2} q^2 + \int \frac{dP}{\rho} \; x(t)$$

Where $x(t)$ is interger constant which is an arbitrary function of t

$$\Rightarrow \quad -\frac{\partial \phi}{\partial t} + \Omega + \frac{1}{2} q^2 + \int \frac{dP}{\rho} = \text{constant}$$

Known as Bernoulli's eq^n for unsteady irratational flows.

—x—